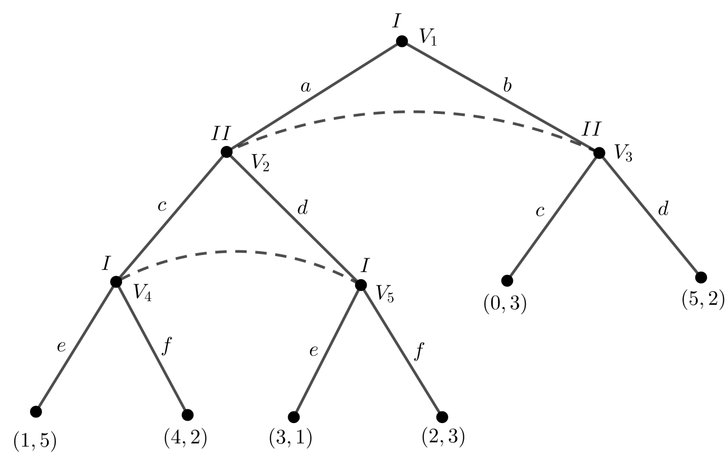
MATH5360 Game Theory
Assignment 4 Q6

6. Consider the game tree



(a) Write down all pure strategies for Player I and Player II.

(b) Write down the strategic form (game bimatrix) of the game.

(c) Solve the subgame after Player I chooses a.

(d) Find all Nash equilibria of the game.

Solution.

(a) Player I: ae, af, be, bf
Player II: c, d

(b) $\begin{pmatrix} (1,5) & (3,1) \\ (4,2) & (2,3) \\ (0,3) & (5,2) \\ (0,3) & (5,2) \end{pmatrix}$

(c) After Player I chooses a, the game bimatrix is

	c	d
e	(1,5)	(3,1)
f	(4,2)	(2,3)

For $\mathbf{x} = (x, 1-x)$ and $\mathbf{y} = (y, 1-y)$, $A\mathbf{y}^T = (3-2y, 2+2y)^T$
$\mathbf{x}B = (2+3x, 3-2x)$
$P = \{(x,y) : (x = 1 \text{ and } 0 \le y < 0.25) \text{ or } (0 \le x \le 1 \text{ and } y = 0.25) \text{ or } (x = 0 \text{ and } 0.25 < y \le 1)\}$
$Q = \{(x,y) : (0 \le x < 0.2 \text{ and } y = 0) \text{ or } (x = 0.2 \text{ and } 0 \le y \le 1) \text{ or } (0.2 < x \le 1 \text{ and } y = 0)\}$
Nash equilibrium after Player I chooses a is $((0.2, 0.8), (0.25, 0.75))$ (Player I uses e, f with probability $0.2, 0.8$ and Player II uses c, d with probability $0.25, 0.75$ respectively).

(d) Suppose Player II uses $bfy = (y, 1-y)$. The payoff to Player I is given by

$$A\mathbf{y}^T = \begin{pmatrix} 1 & 3 \\ 4 & 2 \\ 0 & 5 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} y \\ 1-y \end{pmatrix} = \begin{pmatrix} 3-2y \\ 2+2y \\ 5-5y \\ 5-5y \end{pmatrix}$$

Now if $y < 3/7$ (solving $2+2y = 5-5y$), the best strategy of Player I is *be* or bf. Now by considering the reduced matrix

$$\begin{pmatrix} (4,2) & (2,3) \\ (0,3) & (5,2) \end{pmatrix}$$

we see that the Nash equilibrium of the game is Player I chooses a with a probability 0.5 and *be* or bf with a probability 0.5 and Player II uses $(3/7, 4/7)$ (using c with a probability $3/7$ and d with a probability $4/7$). In the Nash equilibrium, the payoff to Player I and Player II are $20/7$ and 2.5 respectively.